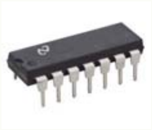
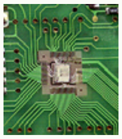
Number Base Conversion

Hardware Evolution

- Computer hardware historically
  - mechanical,
  - electromechanical,
  - electronic
- Integrated circuits vs. Discrete components
- Levels of integration
- SSI, MSI, LSI, VLSI





Integrated Circuit

- An Integrated circuit (IC) is a number of logic gates fabricated on a single silicon chip.
- ICs can be classified according to how many gates they contain as follows:
  - **Small-Scale Integration (SSI):** Contain 1 to 20 gates.
  - **Medium-Scale Integration (MSI):** Contain 20 to 200 gates. Examples: Registers, decoders, counters.
  - **Large-Scale Integration (LSI):** Contain 200 to 200,000 gates. Include small memories, some microprocessors, programmable logic devices.
  - **Very Large-Scale Integration (VLSI):** Usually stated in terms of number of transistors contained usually over 1,000,000. Includes most microprocessors and memories.

Positional Number System Concept

- A number system consists of an order set of symbols (digits) with relations defined for +,-,*, /
- The radix (or base) of the number system is the total number of digits allowed in the number system.
- In positional number systems, a number is represented by a string of digits, where each digit position has an associated weight.
- The value of a number is the weighted sum of the digits.

Decimal Number System

- Radix, r = 10, Digits allowed = radix -1 i.e.

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9

- Number 4321 can be represented in following ways:

1.4000+300+20+1

2.$4*10^3+3*10^2+2*10^1+1*10^0$

- A number with a decimal point is represented by a series of coefficients as follows:

  $a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}\ldots$

Example of Decimal Number

2561.782 can be written as:

$2 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 1 \times 10^0 + 7 \times 10^{-1} + 8 \times 10^{-2} + 2 \times 10^{-3}$

- However, the convention is to write only the coefficients and from their position deduce the necessary powers of 10.

How is data represented in a Computer?

- The digital computer is binary.
- Everything is represented by one of two states:
  - 0, 1
  - on, off
  - true, false
  - voltage, no voltage
- In a computer, values are represented by sequence of binary digits or bits.

Binary Number System

- The binary system is a different number system.
- There are two possible values 0 and 1 known as bits.
- Why Computers use binary?
  - Easy to represent just two things
- If computers used base 10 then they would need to represent 10 things.
- Computers use voltage to represent information.
- With just two levels there is more margin against the noise.
- With 10 levels noise can play havoc with the system.
- (100101) is a binary number.

MSB LSB

Number Systems Used in Computers

Name of Radix	Radix	Set of Digits	Example
Decimal	r=10	{0,1,2,3,4,5,6,7,8,9}	255_{10}
Binary	r=2	{0,1}	11111111_2
Octal	r= 8	{0,1,2,3,4,5,6,7}	377_8
Hexadecimal	r=16	{0,1,2,3,4,5,6,7,8,9,A, B, C, D, E, F}	FF_{16}

Number Systems Used in Computers

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

Bit, Byte and Word

- The smallest unit of storage is the 'Bit' (**Bi**nary Digi**t**)
- The bit has a binary value either '0' or '1'.
- n bits can store numbers from 0 to 2^n
- 'Byte' is a string of bits
- Each memory location is made up of 8 bits called a 'byte'. It is the byte that is addressable.
- A computer with 1MB of memory has 1,048,576 bytes or storage slots (2^{20})
- A "word" is some group of bytes working together -- the actual number depends on the computer or language being used. On a Windows PC, a word is 4 bytes (32 bits), which can represent about 4 billion different values.

Number Base Conversions

- Decimal to Binary
- Binary to Decimal
- Octal to Binary
- Binary to Octal
- Hexadecimal to Binary
- Binary to Hexadecimal
- Decimal to Octal
- Octal to Decimal
- Decimal to Hexadecimal
- Hexadecimal to Decimal

Decimal to Binary Conversion

- Convert 41 from decimal to binary:

2	41	
2	20	1
2	10	0
2	5	0
2	2	1
	1	0

$(41)_{10} = (101001)_2$

Decimal to Binary Conversion

- To convert decimal fractions to binary, repeated multiplication by 2 is used, until the fractional product is 0 (or until the desired number of binary places). The whole digits of the multiplication results produce the answer, with the first as the MSB, and the last as the LSB.
- Example: Convert 0.3125_{10} to binary

			Result Digit
.3125 × 2	=	0.625	0 (MSB)
.625 × 2	=	1.25	1
.25 × 2	=	0.50	0
.5 × 2	=	1.0	1 (LSB)

$(0.3125)_{10} = (0.0101)_2$

Binary to Decimal Conversion

- Remember, each digit represents a power of 2, therefore $(1011)_2$ is

  $1 \bullet 2^3 + 0 \bullet 2^2 + 1 \bullet 2^1 + 1 \bullet 2^0$

  or

  $1 \bullet 8 + 0 \bullet 4 + 1 \bullet 2 + 1 \bullet 1 = 11$

- What about decimal equivalent of $(101.11)_2$?

  $1 \bullet 2^2 + 0 \bullet 2^1 + 1 \bullet 2^0 + 1 \bullet 2^{-1} + 1 \bullet 2^{-2}$

  or

  $1 \bullet 4 + 0 \bullet 2 + 1 \bullet 1 + 0.5 + 0.25 = 5.75$

Octal to Binary Conversion

- Base 8 – uses 0, 1, 2, 3, 4, 5, 6, 7 as digits
- For octal to binary convert each octal digit into its 3 bit binary equivalent. For example:

$(7\ \ 5\ \ 6\ \ 2)_8 = \mathbf{(111101110010)_2}$

111 101 110 010

Binary to Octal Conversion

For binary to octal group each 3-bit starting from least significant bits and convert into one octal digit. For example:

$$(100\,101\,011)_2 = \mathbf{(453)_8}$$

4 5 3

Hexadecimal to Binary Conversion

- Base 16
- Uses 0, 1, 2, 3, 4,5, 6, 7, 8, 9, A, B, C, D, E, F as digits.
- Hexadecimal is indicated by *0x* prefix in computer literature.
- For example 0x2ac in binary will be:

0x2AC

0010 1010 1100

0x2AC = $(001010101100)_2$

Binary to Hexadecimal Conversion

- Just make the group of 4 bits from left to right.
- For example $(101001101111011)_2$ in Hex will be:

0101 0011 0111 1011

↓ ↓ ↓ ↓

5 3 7 B

- So, $(0101\ 0011\ 0111\ 1011)_2$ = *0x*537B

Octal to Decimal Conversion

- The rule is same as we follow in Binary to Decimal conversion.
- But obviously the base is 8
- Example: What is the decimal equivalent of $(725)_8$?

$$(725)_8 = 7x8^2 + 2*8^1 + 5*8^0$$
$$= (448)_{10} + (16)_{10} + (5)_{10}$$
$$= \mathbf{(469)_{10}}$$

Decimal to Octal Conversion

- Convert $(264)_{10}$ to octal

8	264	
8	33	0
	4	1

So, $\mathbf{(264)_{10} = (410)_8}$

Decimal to Hexadecimal Conversion

- Convert $(1128)_{10}$ to Hexadecimal

16	1128	
16	70	8
	4	6

So, $\mathbf{(1128)_{10}}$ = $\mathbf{(468)_{16}}$ or **0x468**

Hexadecimal to Decimal Conversion

- Convert $(FAD1)_{16}$ to decimal

$$
\begin{aligned}
(FAD1)_{16} &= (15)_{10}x16^3 + (10)_{10}x16^2 + (13)_{10}x16^1 + (1)_{10}x16^0 \\
&= (61{,}440)_{10}+(2{,}560)_{10}+(208)_{10}+(1)_{10} \\
&= (64209)_{10}
\end{aligned}
$$